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Improving Timetable Robustness through Optimal Distribution of Runtime Supplement

Bo Jin, Xiaoyun Feng, Qingyuan Wang*, Xiaowen Wang and Cheng Liu

Abstract— **Railway system operations are facing with stochastic disturbances, which cause the delay of trains in operational phase. Thus, a robust timetable should be designed to avoid delay propagation as much as possible in scheduling phase. For that purpose, time allowances are added into timetable in scheduling phase. This paper proposes a mixed integer linear programming (MILP) model that can be used to distribute runtime supplements in order to improve timetable robustness, in which stochastic disturbances are described in the form of probability and integrated into the model. Two simulations based on Guangzhou subway are processed, and sensitivity analysis of the size of total runtime supplement on robustness is observed. The results show that the average delay time can be reduced significantly by applying the proposed MILP method. Meanwhile, this method shows high efficiency in computing the optimal results.**

I. INTRODUCTION

With the increasing scale of railway systems and demand for transportation, railway systems are highly affected by disturbances. Timetable is an important part of railway system management, which determines the position of trains at specific times. Running times between stations, dwell times at stations and headways between trains are all decided in timetable scheduling. Therefore, a robust timetable can help railway system being strong under disturbances, avoiding delay propagation as much as possible. Many researchers have paid attention to robust train timetable planning (RTTP). There are some comprehensive surveys on robust train timetabling [1]-[3].

Methods used to improve timetable robustness can be divided into five major categories [2]. Fischetti and Monaci [4] proposed the concept of *light robustness*, in which slack variables were introduced to relax the feasibility constraints. The sum of the slack variables was minimized to improve timetable robustness. Liebchen et al. [5] applied *recoverable robustness* into RTTP combining robustness and delay management. A set of recovery algorithms were defined to make the timetable recoverable under disturbances. However, only partial recovery actions could be taken into account. *Recovery-to-optimality* was introduced to determine a recovery action which can make the timetable recovered to the scheduling with smallest cost [6]. *Stochastic programming* was a method to improve timetable robustness under pregiven disturbance scenarios. Kroon et al. [7] used a two-stage stochastic model to minimize the weighted delays of trains under real-world disturbance data, where the time supplements and the buffer times are optimized to improve the robustness of cyclic railway timetables. *Lagrangian robustness* applied simple modification into above methods, which performed better in computation efficiency. Cacchiani [8] built a lagrangian heuristic method to improve timetable robustness, which was very competitive and obtained robust solutions of good quality in short computing time.

Either method, the key point of RTTP is to find a more robust timetable by adding time allowances into train process (i.e., running between stations) as runtime supplements, (i.e., departing from starting station) as buffer times, and (i.e., dwelling at stations) as additional dwell times. Meanwhile, the degree of robustness is influenced by the location and size of runtime supplements, buffer times and additional dwell times. Shafia et al. [9] took buffer time as decision variable to optimize timetable under known and unknown distribution functions of disturbances in a single-track railway line. Bešinović et al. [10] evaluated timetable robustness at the macroscopic level, and they adjusted runtime supplements and buffer times iteratively to improve robustness. Meanwhile, the feasibility and stability of timetable were generated at the microscopic level. Şahin [11] solved the RTTP problem by markov chain model, in which departure and arrival times were optimized to improve robustness. Zieger et al. [12] analyzed the impact of different buffer times distribution scheme on delay propagation using an iterative simulation approach. Büker and Seybold [13] used an activity graph to describe the delay propagation, and distribution to describe the disturbance as random variables. Iterative optimization was proposed to form the robust timetable. Solinen et al. [14] focused on the indicator robustness in critical points, and the robustness was evaluated by using the microscopic railway simulation RailSys. Khan and Zhou [15] developed a stochastic programming model for adjusting runtime supplements and additional dwell times that aimed to reduce the average schedule delay. Fischetti et al. [16] proposed four different stochastic programming models focusing on robustness improvement of a given disturbance scenarios. Most of these researches are based on stochastic programming [7], [12]-[16], and optimize timetable iteratively according to its robustness performance in the simulation under stochastic disturbances.

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The authors are with the School of Electrical Engineering (Southwest Jiaotong University), Chengdu 611756, China.
*Corresponding author (e-mail: happyhare32@163.com).

This paper only focuses on stochastic programming for adjusting runtime supplements to improve timetable robustness. The contributions of this paper can be summarized as follows. Different from the typical stochastic programming, stochastic disturbances are described in the form of probability and integrated into the model. The robust timetable can be optimized without iteration, which improve the computational efficiency. Another aspect of the contributions is that a sensitivity analysis of the size of total runtime supplement on robustness is observed.

The rest of this paper is organized as follows. In section II, robust timetable problem and optimization model are stated. In section III, the model of improving timetable robustness is rebuilt into a MILP model. In section IV, case studies of Guangzhou subway are demonstrated to verify the feasibility of the model and the algorithm. Conclusions are given in section V.

II. Robust Timetable Problem

In this section, the influence of runtime supplements on timetable robustness is stated. Meanwhile, optimization model for RTTP is built under several necessary assumptions to reduce average delay time.

A. Problem Description

A train timetable is identified as robust when it can avoid delay propagation as much as possible. An effective way to avoid delay propagation is adding runtime supplement into train running process between stations, which can absorb potential delay occurring in practical operation. When affected by disturbance, a train will departure from the station later than the pregiven departure time, if the initial delay time is shorter than the runtime supplement of adjacent interstation, then the train can arrive at the next station punctually. However, if the disturbance is very strong, delay will propagate to the next stations. Obviously, with more runtime supplements adding to running process between stations, timetable will perform better at robustness. Total runtime time of the travel will

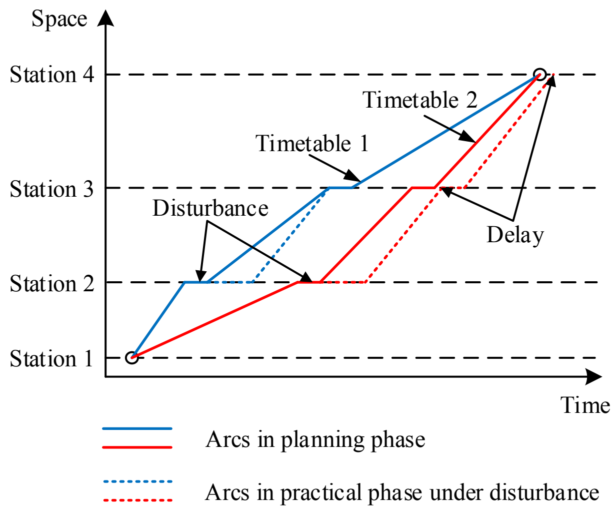

Figure 1. The delay propagation of two timetables

increase when more runtime supplements are added, which will reduce the transportation efficiency of timetable. Therefore, the size of runtime supplement is limited in order to maintain efficiency in the timetable planning phase. On the other hand, the size and position of runtime supplement addition will influence the robustness. Fig. 1 shows the delay propagation phenomenon of two timetables with different runtime supplement distribution scheme. When a disturbance take place in Station 2, the Timetable 1 can absorb the influence of disturbance and avoid delay propagation with the help of sufficient runtime supplement in the interstation between Station 2 and Station 3, the Timetable 2 can't avoid delay propagation effectively, delay propagate to Station 3 and Station 4. Therefore, distributing runtime supplements (size and position) wisely can improve timetable robustness.

In addition, the position and degree of disturbances will influence the distribution of runtime supplements. If the disturbance take place in Station 3 in Fig. 1, more runtime supplement should be added in the interstation between Station 2 and Station 3 to reduce delay propagation. However, the state of disturbances is difficult to define in planning phase. In this paper, the state of disturbances is defined according to a lot number of actual operational data, and three key indicators are defined, including position, original delay time and probability. Then, the distribution of runtime supplements can be optimized under the pretreated disturbances, which means that disturbances are expressed by position, intensity and probability. Meanwhile, the average delay time of stations along a line is set as the key performance indicator of timetable robustness.

B. Assumptions and Notations

In order to clarify main purpose of this paper, the following assumptions are stated:

1) Orders of trains do not change in the operational phase. Trains have to operate according to the timetable, and overtaking is not allowed in this paper.

2) Adding or removing a train is not allowed. In other words, disturbance that affects normal operation is not discussed in this paper.

3) Disturbances will not happen at the same time. The delay is caused by single disturbance, not by multiple disturbances jointly. Disturbances are taken into account only after the previous delay propagation is over.

4) The dwell times are fixed in the operational phase, which is to ensure passengers get on and off the train. The optimization of additional dwell time is not considered in this paper.

5) Delay propagation will not influence adjacent trains, because of that the internal delay time is shorter than the pregiven buffer times. The optimization of buffer times is not considered in this paper.

Next, notions used throughout the remainder of this paper are introduced as followings:

1) Decision Variables:

t_i runtime supplement of the ith interstation.

2) Parameters:

N number of interstations;

K number of disturbances;

T_t total runtime;

$T_{\min}$ minimum total runtime;

$T_{\sup}$ total runtime supplement;

T_d average delay time;

$t_{i,\min}$ minimum runtime supplement of the ith interstation;

$t_{i,\max}$ maximum runtime supplement of the ith interstation;

I^k original intensity of the kth disturbanc;

S^k station num of the kth disturbance happening at;

P^k probability of the kth disturbance, which is between 0 and 1;

ε a small positive number;

$d^k_{\max}$ maximum delay time of the kth disturbance, which is equal to I^k;

$d^k_{\min}$ minimum delay time of the kth disturbance, which is equal to 0.

3) Intermediate Variables:

d_n^k delay time propagating to the nth station of the kth disturbance;

δ_n^k logical variable corresponding to d_n^k;

$\eta^k_{n,i}$ auxiliary variable corresponding to the product $\delta_n^k t_i$;

Additionally, disturbances happening in the interstation can be treated as disturbances happening in the next adjacent station, and the original delay time is equal to the delay time of arriving at the next adjacent station.

C. Model Formulation

The process of a train traveling in a line from Station 1 to Station N+1 is researched, which includes N interstations. Firstly, the total runtime of the travel consists of the minimum total runtime and the total runtime supplement, which can be described as:

$$T_t = T_{\min} + T_{\sup} \tag{1}$$

The total runtime supplement satisfies the constraint:

$$T_{\sup} = \sum_{i=1}^{N} t_i \tag{2}$$

And, the runtime of each interstations should satisfies the constraint:

$$t_{i,\min} \le t_i \le t_{i,\max},\ 1 \le i \le N \tag{3}$$

Avoiding delay propagation is a major indicator evaluating timetable robustness. Reducing the average delay time of stations is an effective way to improve the ability of avoiding delay propagation. Thus, the objective function of improving timetable robustness model is described as:

$$\min\ T_d = \sum_{k=1}^{K}\left(P^k \sum_{n=1}^{N+1} d_n^k\right) \tag{4}$$

The delay time d_n^k is influenced by the position of disturbance and station. According to the positional relationship between the disturbance and station, the calculation of delay time d_n^k causing by the kth disturbance can be divided into three conditions:

1) the nth station is before the S^kth station:

$$d_n^k = 0 \text{ for } n < S^k \tag{5}$$

2) the nth station is the S^kth station:

$$d_n^k = I^k \text{ for } n = S^k \tag{6}$$

3) the nth station is after the S^kth station:

$$d_n^k = I^k - \sum_{i=S^k}^{n-1} t_i \text{ for } n > S^k \tag{7}$$

However, delay time must be positive, delay time of the condition *1)* has no practical meaning, delay time of the condition *3)* could be negative, which requires further explanation. The calculation of the condition *3)* can be concluded as:

$$d_n^k\left(n > S^k\right) = \begin{cases} I^k - \sum_{i=S^k}^{n-1} t_i & I^k - \sum_{i=S^k}^{n-1} t_i > 0 \\ 0 & I^k - \sum_{i=S^k}^{n-1} t_i \le 0 \end{cases} \tag{8}$$

In this model, runtime supplement is taken as decision variables, the optimal objective is to reduce the average delay time. The constraints are all linear equations. The objective function is nonlinear because of the complex calculation of delay time, which improves the difficulty of solving.

III. The MILP Problem

In this section, logical and auxiliary variables are introduced to linearize the calculation of delay time, which can make the optimal model rebuilt into a MILP model.

A. Logical Variables

In order to build MILP model, the calculation of delay time d_n^k should be transformed into linear equations. Here, logical variable δ_n^k is introduced according to the three conditions, defined as:

1) the nth station is before the S^kth station:

$$\delta_n^k = 0 \text{ for } n < S^k \tag{9}$$

2) the nth station is the S^kth station:

$$\delta_n^k = 1 \text{ for } n = S^k \tag{10}$$

3) the nth station is after the S^kth station:

$$\begin{cases} \left[I^k - \sum_{i=S^k}^{n-1} t_i > 0 \right] \Leftrightarrow \left[\delta_n^k = 1 \right] \\ \left[I^k - \sum_{i=S^k}^{n-1} t_i \le 0 \right] \Leftrightarrow \left[\delta_n^k = 0 \right] \end{cases} \text{for } n > S^k \tag{11}$$

Since the maximum and minimum values of d_n^k are $d^k_{\max}$ and $d^k_{\min}$ respectively, the logical conditions (11) can be expressed as [17]:

$$\begin{cases} d_n^k \le \delta_n^k d_{\max}^k \\ d_{\min}^k - d_n^k \le \delta_n^k \left(d_{\min}^k - \varepsilon \right) \end{cases} \tag{12}$$

Where, ε is introduced to transform a strict equality into a non-strict inequality, which fits the MILP framework. The function (8) can be rewritten as:

$$d_n^k = \delta_n^k \left(I^k - \sum_{i=S^k}^{n-1} t_i \right) \text{ for } n > S^k \tag{13}$$

Then, with the help of logical variable δ_n^k, the calculation of delay time can be described by $\delta_n^k d_n^k$. The objective function (4) can be rewritten as:

$$\begin{aligned} \min\ T_d &= \sum_{k=1}^{K} \left(P^k \sum_{n=1}^{N+1} \delta_n^k d_n^k \right) \\ &= \sum_{k=1}^{K} \left\{ P^k I^k + P^k \sum_{n=S^k+1}^{N+1} \left[\delta_n^k \left(I^k - \sum_{i=S^k}^{n-1} t_i \right) \right] \right\} \\ &= \sum_{k=1}^{K} \left[P^k I^k + P^k \sum_{n=S^k+1}^{N+1} \left(\delta_n^k I^k - \sum_{i=S^k}^{n-1} \delta_n^k t_i \right) \right] \end{aligned} \tag{14}$$

Where, the product $\delta_n^k t_i$ is nonlinear, which doesn't fit the MILP framework. Therefore, auxiliary variables are introduced to solve this problem.

B. Auxiliary Variables

In order to build MILP model, the nonlinear product $\delta_n^k t_i$ in function (14) needs to be linearized. Auxiliary variable $\eta^k_{n,i}$ is introduced, defined as:

$$\eta_{n,i}^k = \delta_n^k t_i \tag{15}$$

Since the maximum and minimum values of t_i are $t_{i,\min}$ and $t_{i,\max}$ respectively, the function (15) can be expressed as [17]:

$$\begin{cases} \eta_{n,i}^k \le t_{i,\max} \delta_n^k \\ \eta_{n,i}^k \ge t_{i,\min} \delta_n^k \\ \eta_{n,i}^k \le t_i - t_{i,\min} \left(1 - \delta_n^k \right) \\ \eta_{n,i}^k \ge t_i - t_{i,\max} \left(1 - \delta_n^k \right) \end{cases} \tag{16}$$

Then, the objective function (14) can be rewritten a linear function as:

$$\begin{aligned} \min\ T_d &= \sum_{k=1}^{K} \left[P^k I^k + P^k \sum_{n=S^k+1}^{N+1} \left(\delta_n^k I^k - \sum_{i=S^k}^{n-1} \eta_{n,i}^k \right) \right] \\ &= \sum_{k=1}^{K} P^k I^k + \sum_{k=1}^{K} \sum_{n=S^k+1}^{N+1} P^k \delta_n^k I^k - \sum_{k=1}^{K} \sum_{n=S^k+1}^{N+1} \sum_{i=S^k}^{n-1} P^k \eta_{n,i}^k \end{aligned} \tag{17}$$

C. The MILP Model

After introducing logical and auxiliary variables, the model in the section II can be rebuilt into a MILP model. The decision variables of the MILP model can be defined as:

$$\begin{cases} \tilde{t} = \left[t_1 \cdots t_N \right] \\ \delta = \left[\delta_1^1 \cdots \delta_{N+1}^1 \cdots \delta_1^K \cdots \delta_{N+1}^K \right] \\ \eta = \left[\eta^1 \cdots \eta^k \cdots \eta^K \right] \end{cases} \tag{18}$$

Where, variable η^k can be described as:

$$\eta^k = \left[\eta_{1,1}^k \cdots \eta_{N+1,1}^k \cdots \eta_{1,N}^k \cdots \eta_{N+1,N}^k \right] \tag{19}$$

And, some of decision variables are binary and some are real variables, decision variable matrix of the MILP model is described as:

$$X = \left[\tilde{t} \quad \delta \quad \eta \right]^T \tag{20}$$

And, the objective function of the MILP model can be described as:

$$\min\ T_d = F \bullet X \tag{21}$$

subject to

$$M_1 \bullet X \le m_1 \tag{22}$$

$$M_2 \bullet X = m_2 \tag{23}$$

Where, the coefficient matrix F can be obtained according to the function (17), described as:

$$F = \left[0 \cdots 0 \cdots P^1 I^1 \cdots P^K I^K \cdots P^1 \cdots P^K \right] \tag{24}$$

And, the coefficient matrix M_1 and m_1 can be obtained according to the function (3), (12) and (16). The coefficient matrix M_2 and m_2 can be obtained according to the function (2).

In addition, the MILP problem (21)-(23) can be solved by branch-and-bound algorithms implemented in several existing commercial and free solvers. In this paper, we use CPLEX to solve this problem.

IV. CASE STUDIES

In this section, proposed method is applied to a metro railway instance of peak hours and off-peak hours. For disturbances in different scenarios, different distribution schemes of runtime supplement are compared and sensitivity of the size of total runtime supplement on robustness is analyzed. Meanwhile, we discuss the impact of total runtime on average delay time. Case studies are based on a metro line in Guangzhou, which consists of 13 stations. Disturbance data comes from actual operation, which is represented in an appropriate form. Case studies are tested under MATLAB environment on a computer with an Intel Core i5 2.30 GHz CPU and 8GB RAM, which are performed using CPLEX Slover 12.6.

A. Simulation of Off-peak Hours

The total runtime of off-peak hours is 1126s, the minimum runtime is 994s, and the total runtime supplement is 132s. The minimum runtime supplement of each interstation is 9s, and the maximum runtime supplement is 14s. Disturbances of peak hours are shown in TABLE I.

Three kinds of disturbance scheme are compared in this simulation:

- **PDS**: Practical Distribution Scheme.
- **EDS**: Equal Distribution Scheme.
- **ODS**: Optimal Distribution Scheme.

Three disturbance schemes of off-peak hours are shown in TABLE II. As shown in TABLE V. the average delay times of PDS and EDS are 39.6s and 38.3s respectively. The average delay time of ODS is 33.3s, which is 15.9% lower than PDS and 13.1% lower than EDS. The computing time of ODS is 0.09s, which reflects the high computational efficiency of the method.

TABLE I. DISTURBANCES OF OFF-PEAK HOURS

Num	Position	Intensity (s)	Probability
1	Station 1	20	1/9
2	Station 5	20	1/9
3	Station 8	30	3/9
4	Station 11	20	2/9
5	Station 12	20	2/9

TABLE II. RUNTIME DISTRIBUTION SCHEMES OF OFF-PEAK HOURS

Num of interstation	Runtime (s)		
	PDS	EDS	ODS
1	14	11	14
2	9	11	10
3	10	11	6
4	12	11	6
5	12	11	14
6	9	11	12
7	14	11	8
8	9	11	14
9	10	11	14
10	11	11	6
11	13	11	14
12	9	11	14

B. Simulation of Peak Hours

In this part, the simulation of peak hours is processed. The total runtime of peak hours is 1066s, the minimum runtime is 994s, and the total runtime supplement is 72s. The minimum runtime supplement of each interstation is 4s, and the maximum runtime supplement is 12s. Disturbances of peak hours are shown in TABLE III.

Three kinds of distribution scheme are compared in this simulation. Runtime supplements of PDS, EDS and ODS are shown in TABLE IV. As shown in TABLE V. the average delay times of PDS and EDS are 79.1s and 78.1s respectively. The average delay time of ODS is 70.3s, which is 11.1% lower than PDS and 9.9% lower than EDS. Without changing the total runtime, the ODS can improve the timetable robustness obviously. The computing time of ODS is 0.13s. Compared with the simulation of off-peak hours, with the increasing of disturbances (quantity and intensity), the average delay time of this simulation has increased obviously, which reflects that there is more room for improvement of timetable robustness in peak hour. Additionally, the computing time has not increased a lot, which reflects that the method can respond to huge-amount disturbance scenarios. In order to prove computing efficiency, a simulation with 50 different disturbances is processed and the ODS can be calculated in 1.41s, which is not described here.

TABLE III. DISTURBANCES OF PEAK HOURS

Num	Position	Intensity (s)	Probability
1	Station 1	20	1/14
2	Station 4	30	2/14
3	Station 5	20	3/14
4	Station 8	40	2/14
5	Station 10	60	1/14
6	Station 11	20	3/14
7	Station 12	30	2/14

TABLE IV. RUNTIME DISTRIBUTION SCHEMES OF PEAK HOURS

Num of interstation	Runtime (s)		
	PDS	EDS	ODS
1	10	6	4
2	4	6	4
3	5	6	4
4	6	6	4
5	6	6	12
6	4	6	4
7	10	6	4
8	4	6	12
9	5	6	4
10	6	6	4
11	8	6	12
12	4	6	4

TABLE V. AVERAGE DELAY TIMES

Period	Average delay time (s)		
	PDS	EDS	ODS
Off-peak	39.6	38.3	33.3
Peak	79.1	78.1	70.3

C. Sensitivity Analysis of the Size of Total Runtime Supplement on Robustness

In order to analyze the sensitivity of the size of total runtime supplement on timetable robustness, runtime supplements are added into the simulation of off-peak hours and peak hours respectively. The trend of the average delay times is shown in Fig 1. The average delay times can be efficiently reduced by increasing the total runtime supplement. However, when the total runtime supplement reaches a certain level, the effect is in vain. The time thresholds of simulation of off-peak hours and peak hours are both 120s. Adding runtime supplement above 120s has no effect on reducing the average delay time. Therefore, runtime supplement should be added based on the relationship between the total runtime supplement and the average delay time to make runtime supplement play a better role in improving timetable robustness.

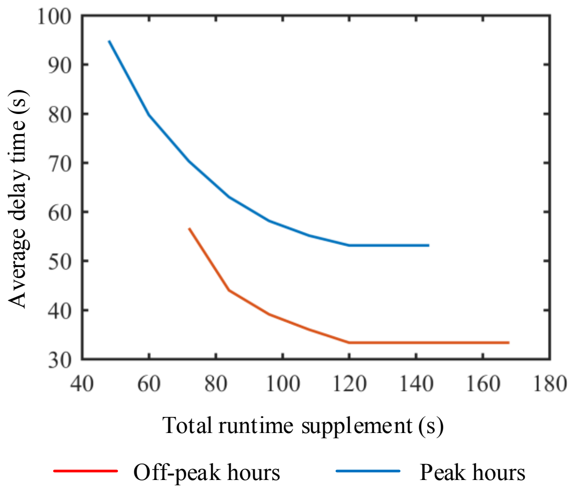

Figure 2. Relationship between the total runtime supplement and the average delay time

V. Conclusion

For improving timetable robustness, a MILP model is built to reduce the average delay time by optimizing the distribution scheme of runtime supplements. The calculation of delay times is analyzed, which is linearized by introducing logical and auxiliary variables. Two simulations based on Guangzhou subway (off-peak and peak hours) are processed. Simulation results show that the average delay time can be reduced by 15.3% and 11.1% for off-peak and peak hours respectively. Additionally, the high computing efficiency of this method supports for its application to robust train timetable planning (RTTP) with larger scale and more complex disturbance scenarios. The research on the relationship between the total runtime supplement and the average delay time can be applied to problem about optimizing runtime. Timetable robustness can be considered in the problem of multi-objective optimization.

As the future work, more timetable parameters, like buffer time and additional dwell time, should be considered into RTTP to make a more robust timetable. The disturbances that the model consider should be more general, which can make the timetable performances more stable facing with various disturbances. In addition, solutions that make the timetable recoverable under disturbances should be researched. Different solutions can be proposed for different disturbances.

References

[1] V. Cacchiani and P. Toth, "Nominal and robust train timetabling problems," *Eur. J. Oper. Res.*, vol. 268, no. 3, pp. 93–115, 2012.

[2] V. Cacchiani and P. Toth, "Robust Train Timetabling," in *Handbook of Optimization in the Railway Industry*, 2018, pp. 93–115.

[3] R. M. Lusby, J. Larsen, and S. Bull, "A survey on robustness in railway planning," *Eur. J. Oper. Res.*, vol. 266, no. 1, pp. 1–15, 2018.

[4] M. Fischetti and M. Monaci, "Light robustness," in *Robust and Online Large-Scale Optimization*, New York: Springer-Verlag, 2009, pp. 61–84.

[5] C. Liebchen, M. Lübbecke, R. Möhring, and S. Stiller, "The concept of recover- able robustness, linear programming recovery, and railway applications," in *Ro- bust and online large-scale optimization*, 2009, pp. 1–27.

[6] M. Goerigk and A. Schöbel, "Recovery-to-optimality: A new two-stage approach to robustness with an application to aperiodic timetabling," *Comput. Oper. Res.*, vol. 52, no. PART A, pp. 1–15, 2014.

[7] L. Kroon, G. Maróti, M. R. Helmrich, M. Vromans, and R. Dekker, "Stochastic improvement of cyclic railway timetables," *Transp. Res. Part B Methodol.*, vol. 42, no. 6, pp. 553–570, 2008.

[8] V. Cacchiani, A. Caprara, and M. Fischetti, "A Lagrangian Heuristic for Robustness, with an Application to Train Timetabling," *Transp. Sci.*, vol. 46, no. 1, pp. 124–133, 2012.

[9] M. A. Shafia, M. Pourseyed Aghaee, S. J. Sadjadi, and A. Jamili, "Robust train timetabling problem: Mathematical model and branch and bound algorithm," *IEEE Trans. Intell. Transp. Syst.*, vol. 13, no. 1, pp. 307–317, 2012.

[10] N. Bešinović, R. M. P. Goverde, E. Quaglietta, and R. Roberti, "An integrated micro-macro approach to robust railway timetabling," *Transp. Res. Part B Methodol.*, vol. 87, pp. 14–32, 2016.

[11] İ. Şahin, "Markov chain model for delay distribution in train schedules: Assessing the effectiveness of time allowances," *J. Rail Transp. Plan. Manag.*, vol. 7, no. 3, pp. 101–113, 2017.

[12] S. Zieger, N. Weik, and N. Nießen, "The influence of buffer time distributions in delay propagation modelling of railway networks," *J. Rail Transp. Plan. Manag.*, vol. 8, no. 3–4, pp. 220–232, 2018.

[13] T. Büker and B. Seybold, "Stochastic modelling of delay propagation in large networks," *J. Rail Transp. Plan. Manag.*, vol. 2, pp. 34–50, 2012.

[14] E. Solinen, G. Nicholson, and A. Peterson, "A microscopic evaluation of railway timetable robustness and critical points," *J. Rail Transp. Plan. Manag.*, vol. 7, no. 4, pp. 207–223, 2017.

[15] M. B. Khan and X. Zhou, "Stochastic optimization model and solution algorithm for robust double-track train-timetabling problem," *IEEE Trans. Intell. Transp. Syst.*, vol. 11, no. 1, pp. 81–89, 2010.

[16] M. Fischetti, D. Salvagnin, and A. Zanette, "Fast Approaches to Improve the Robustness of a Railway Timetable," *Transp. Sci.*, vol. 43, no. 3, pp. 321–335, 2009.

[17] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.